Reading pH and Logarithmic Scales

The pH scale displays H^+ ion concentration in a way that lets us see important differences in the very small numbers which are encountered in the lab and in the everyday environment. The table below shows typical [H+] and pH values for common solutions:

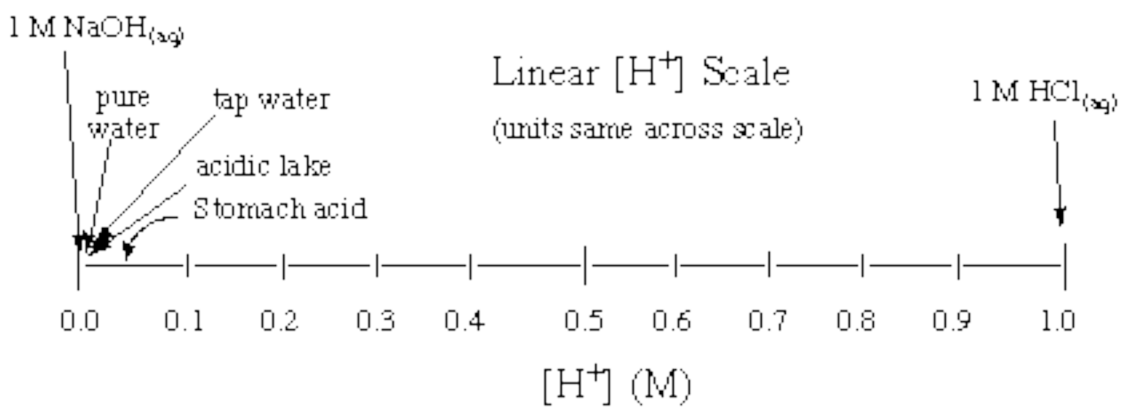
Solution or mixture	$[H^+]$ (M)	$[HO^-]$ (M)	pH
1 M hydrochloric acid	1.0	1.0×10^{-14}	0
Stomach acid	$\sim 4 \times 10^{-2}$	2.5×10^{-13}	1.4
Water in acid lake (fish dying)	$\sim 2 \times 10^{-5}$	$\sim 5 \times 10^{-10}$	4.7
Tap water	$\sim 4 \times 10^{-6}$	$\sim 2.5 \times 10^{-9}$	5.4
Pure water	1.0×10^{-7}	1.0×10^{-7}	7
Baking soda solution ($NaHCO_3$)	~ 1 x $\times 10^{-9}$	$\sim 1 \times 10^{-5}$	9
Drano drain cleaner [$Al(OH)_3$]	$\sim 1 \times 10^{-13}$	~ 0.1	13
1 M sodium hydroxide	1.0×10^{-14}	1.0	14

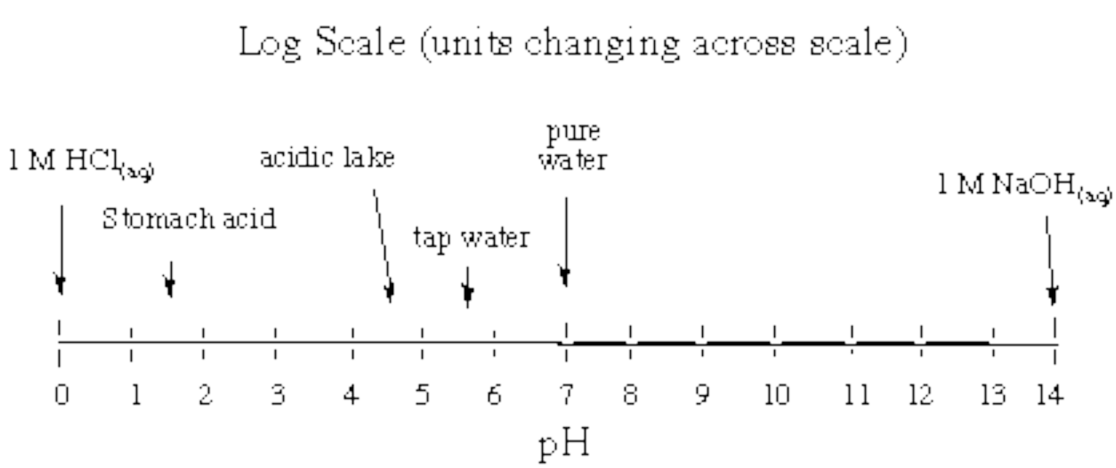
Multiply the concentrations of H^+ and HO^- on each line and you'll see that the product is always 1.0×10^{-14} M^2. This turns out to always be the case in water solution so this *water dissociation constant* can be used to determine $[HO^-]$ if you only know $[H^+]$

$$K_w = [H^+][HO^-] = 1.0 \times 10^{-14}$$

$$\text{so } [H^+] = K_w \div [HO^-] \quad \text{and} \quad [HO^-] = K_w \div [H^+]$$

Logarithms (we are only using $\log_{10}$ here) are used to spread out small numbers so they can be displayed on graphs and so that we can talk about actual numbers in a way that is easier to follow. To see why this is necessary, let's show where the data from the table appear on a linear and a log scale:





You see it's easier to tell differences using pH values than molar concentrations when concentrations are small, ***and they usually are small***.

pH is defined as follows: $pH = -\log_{10}[H^+]$

(More on logarithms below). This means that if you have $[H^+]$ concentrations of :

$[H^+] = 0.1$ M $\quad$ $pH = -\log_{10}(0.1) = -(-1) = 1$ $\quad$ $[0.1 = 10^{-1}$, so $\log_{10}(0.1) = \mathbf{-1}$
$[H^+] = 0.0000001 = 10^{-7}$ $\quad$ $pH = -\log_{10}(10^{-7}) = -(-1) = \mathbf{7}$

There are a few key things to understand about pH and reading log scales:

1. We use $[H^+]$ to do the calculation whether a solution is acidic or basic. It is done this way in order to have a common reference for all solutions.

2. pH values are ***negative logarithms*** of $[H^+]$. A common (base 10) log is the exponent to which you have to raise 10 to get a particular number:

   100 is 10^2, so $\log_{10}(100) = 2$ $\qquad$ 0.00010 is 10^{-4} so $\log_{10}(0.00010) = -4$

   numbers in-between powers of 10 are decimal fractions, such as:

   $2 = 10^{0.301}$ so $\log_{10}(2) = 0.301$

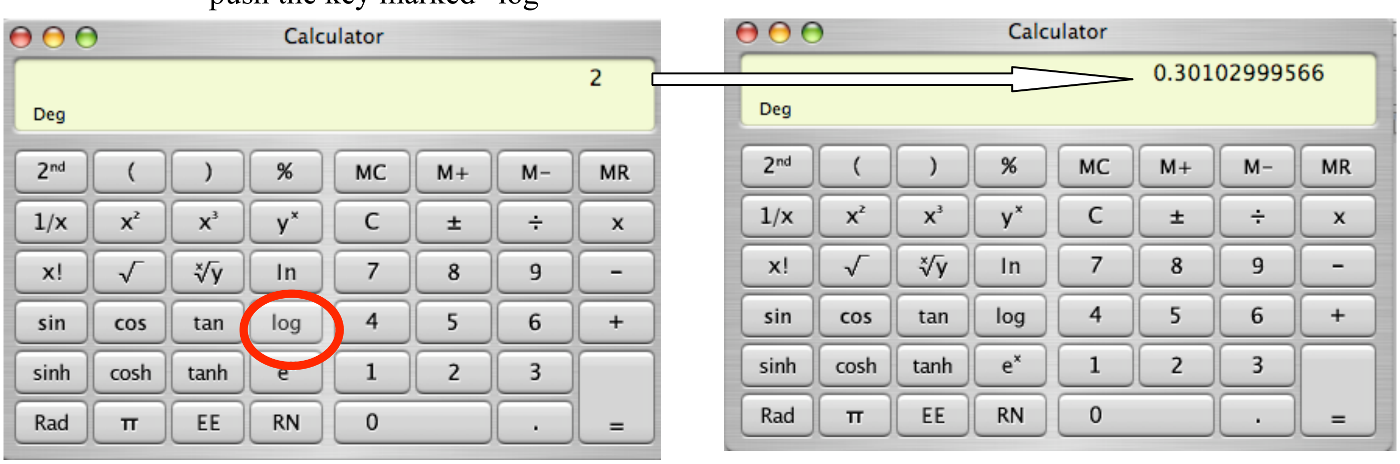
   Also, logs add, for example: $\log_{10}[2 \times 10^{-5}] = \log_{10}[2] + \log_{10}[10^{-5}] = 0.301 + (-5) = -4.69$

   This is true for the same reason that you add exponents when you multiply numbers

   $1000 \times 1000 = 1$ million is the same as $10^3 \times 10^3 = 10^{(3+3)} = 10^6$

3. Since there is a minus sign in the definition of pH,
   SMALLER pH NUMBERS CORRESPOND TO LARGER H^+ CONCENTRATIONS.

4. Any scientific calculator will convert a number to its log as follows:
   - Enter the number you want to take a log of (for pH calculations, this is [H+])
   - push the key marked "log"



Type in 2, press "log" and you see 0.30102999566

(you usually round to 2 digits past the decimal – in this case 0.30 - for pH calculations)

Try taking logs of the following numbers: 10,000 200, 0.002 and 5×10^{-5}
(The pH values corresponding to the last two are 2.70 and 4.301)

$Log_{10}(10000)$ = ________________ $Log_{10}(200)$ = ________________

$Log_{10}(.002)$ = ________________ $Log_{10}(5 \times 10^{-5})$ = ________________

In the future, you will need to remember the trends: high pH numbers (>7) mean basic solutions ($[HO^-] > [H^+]$) and low pH numbers (<7) mean acidic solutions. pH 7 is neutral. These three points are worth memorizing because they are part of the common technical language used throughout the sciences, medicine and engineering.

The pH scale may seem like a complicated and kind of backwards way of displaying numbers but it helps a lot to keep real-world values straight. Think about how hard it would be to understand which number is larger when someone says: "This sample has four times ten to the minus-fifth molar H ion concentration and that sample has two times ten to the minus-seventh". You are very likely is to only hear the coefficients (2 and 4) or the powers of 10 (-5 and -7) and you may forget to include the minus sign and think that the 10^{-7} term is bigger than the 10^{-5} term. If you need to know pH values of solutions in your future work, you will develop a mental calibration where if you are told, for example, something is pH 6, you immediately register that this is somewhat acidic (pH < 7, but $[H^+]$ is within a factor of 10 of neutral), and if you hear pH 14, you know this is a very basic solution.

For the remaining exams, you will only be expected to recognize trends (lower pH means higher acid & vice versa) and calculate pH from the general equation.

Other information we get is also displayed on log scales. The Richter scale for gauging the strength of earthquakes displays how large the ground movement is on a log scale. Each unit increase on the Richter scale is a x 10 change, so a magnitude 5 earthquake (usually no injuries and minimal building damage) is 100 times (10^2 or 2 log_{10} units) weaker than a magnitude 7.0 quake (bottom of the "major" earthquake range - may give serious injuries and/or deaths depending on location and building stability), which is ten times weaker than a 8.0 (upper limit of the "major" earthquake range – at and above 8.0 we start saying a quake is "catastrophic").